A neutron star model in the nonlinear Relativistic Mean-Field Theory

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The neutron star parameters in the model extended by the inclusion of δ meson and additional nonlinear vector meson interactions are studied.

The relativistic nonlinear mean field model

The aim of this paper is to study the influence of the additional δ meson and nonlinear vector meson interactions on neutron star parameters. The considered model comprises: nucleons, mesons and leptons. The starting point is the construction of the effective Lagrangian intended for application to the model described

$$\begin{aligned}\mathcal{L} \quad &= \tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - U(\varphi) - \tfrac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \tfrac{1}{2}M_\omega^2\omega_\mu\omega^\mu + \tfrac{1}{4}c_3(\omega_\mu\omega^\mu)^2 + \\ &-\tfrac{1}{4}R^a_{\mu\nu}R^{a\mu\nu} + \tfrac{1}{2}M_\rho^2 b^a_\mu b^{a\mu} + \tfrac{1}{4!}g_\rho^4\zeta(b^a_\mu b^{a\mu})^2 + \tfrac{1}{2}\partial_\mu\delta^a\partial^\mu\delta^a - \tfrac{1}{2}M_\delta^2\delta^a\delta^a \\ &+(g_\rho g_\omega)^2\Lambda_v b^a_\mu b^{a\mu}\omega_\mu\omega^\mu + (g_\rho g_s)^2\Lambda_4 b^a_\mu b^{a\mu}\varphi^2 + \\ &i\overline{\psi}\gamma^\mu D_\mu\psi - \overline{\psi}(M - g_s\varphi - I_{3N}g_\delta\tau^a\delta^a)\psi.\end{aligned} \tag{1}$$

The scalar sector consists of two isoscalar (σ and ω) and two isovector (ρ and δ) mesons, denoted by φ, ω_μ, b^a_μ and δ^a respectively. Meson δ has to be included as the model describes highly asymmetric neutron star matter. $R^a_{\mu\nu}$, $\Omega_{\mu\nu}$, $F_{\mu\nu}$ are the field tensors, the covariant derivative is given by $D_\mu = \partial_\mu + 1/2ig_\rho b^a_\mu\tau^a + ig_\omega\omega_\mu$. The potential function has the form

$$U(\varphi) = 1/2m_s^2\varphi^2 + 1/3g_2\varphi^3 + 1/4g_3\varphi^4. \tag{2}$$

The nucleon masses are denoted by M_N whereas m_s, M_ω, M_ρ and M_δ are masses assigned to the meson fields.
The parameters entering the Lagrangian function (1) have been chosen to reproduce the equilibrium properties of symmetric nuclear matter. As it is usually assumed in quantum hadrodynamics [1] the mean field approximation is adopted and for the ground state of homogeneous infinite matter, quantum fields operators are replaced by their classical expectation values. Thus mesonic fields can be separated into classical mean field values

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and quantum fluctuations which are not included in the ground state:

$$\begin{array}{ll} \sigma = \overline{\sigma} + s & \delta^a = \overline{\delta}^a + d\delta^{3a} \\ \omega_\mu = \overline{\omega}_\mu + w_0\delta_{\mu 0} & b^a_\mu = \overline{b}^a_\mu + r_0\delta_{\mu 0}\delta^{3a} \end{array} \tag{3}$$

The field equations derived from the Lagrangian function at the mean field level are

$$(m_s^2 - 2(g_\rho g_s)^2\Lambda_4 r_0^2)s + g_3 s^2 + g_4 s^2 = \sum_N g_s M^2_{eff,N,k_F} S(M_{eff,N}, k_{F,N}) \tag{4}$$

$$(m_\omega^2 + 2(g_\rho g_\omega)^2\Lambda_v r_0^2)w_0 + c_3 w_0^3 = n_B \tag{5}$$

$$(m_\rho^2 + 2(g_\rho g_s)^2\Lambda_4 s^2)r_0 + 2(g_\rho g_\omega)^2\Lambda_v r_0 w_0^2 + \frac{1}{6}g_\rho^4\zeta r_0^3 = g_\rho I_{3N} n_B \tag{6}$$

$$m_\delta^2 d^3 = \sum_N g_{\delta N} I_{3N} S(M_{eff,N}, k_{F,N}). \tag{7}$$

Their forms indicate the in-medium modification of meson and nucleon masses [2],[3]. The function $S(M_{eff,N}, k_{F,N})$ is expressed with the use of Fermi integrals

$$S(M_{eff,N}, k_{F,N}) = \frac{2J_N + 1}{2\pi^2}\int_0^{k_{FN}} \frac{M_{N,eff}}{\sqrt{k^2 + M_{N,eff}}} k^2 dk \tag{8}$$

where J_N and I_{3N} are the spin and isospin projection, $k_{F,N}$ is the Fermi momentum of species N ($N = n, p$), n_B denotes the baryon density. The effective nucleon mass which follows from the Dirac equation has the form

$$M_{N,eff} = M_N - g_s s + I_{3N} g_\delta d \tag{9}$$

Table 1

Parameters of this model

parameter set	g_δ	g_ρ	ζ	Λ_v	Λ_4
TM1	9.264	0	0.	0	0
GM3	10.5	5.25	0	0	0
nonl. TM1	3.5	10.9	0.5	0.008	0.001

The main effect of the inclusion of δ meson and nonlinear vector meson interactions becomes evident studying properties of neutron star matter, especially the form of the energy density ε and the equation of state. These two equations include contributions coming from meson, lepton and nucleon fields and are given in the following forms

$$\begin{aligned} P &= \frac{1}{2}m_\rho r_0^2 + \frac{1}{2}m_\omega w_0^2 + \frac{1}{4}c_3 w_0^4 - \frac{1}{2}m_\delta d^2 + \Lambda_v(g_\rho g_\omega)^2 r_0^2 w_0^2 \\ &+ \Lambda_4(g_\rho g_s)^2 r_0^2 s^2 + \frac{1}{24}\zeta g_\rho^4 r_0^4 - U(s) + P_N + P_L \end{aligned} \tag{10}$$

$$\begin{aligned} \varepsilon &= \frac{1}{2}m_\rho^2(r_0)^2 + \frac{1}{2}m_\delta d^2 + 3\Lambda_v(g_\rho g_\omega)^2 r_0^2 w_0^2 + \Lambda_4(g_\rho g_s)^2 r_0^2 s^2 \\ &+ \frac{1}{2}m_\omega^2 w_0^2 + \frac{3}{4}c_3 w_0^4 + \frac{1}{8}\zeta g_\rho^4 r_0^4 + U(s) + \epsilon_N + \epsilon_L. \end{aligned} \tag{11}$$

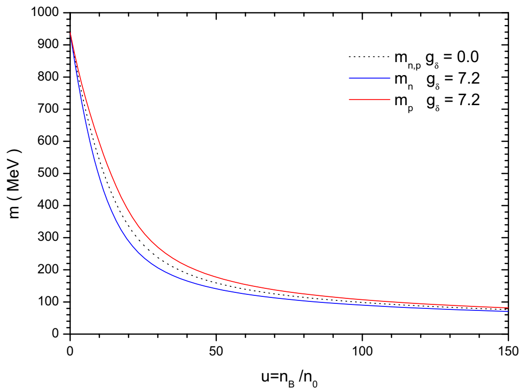

Figure 1. The nucleon effective masses in the nonlinear RMF theory.

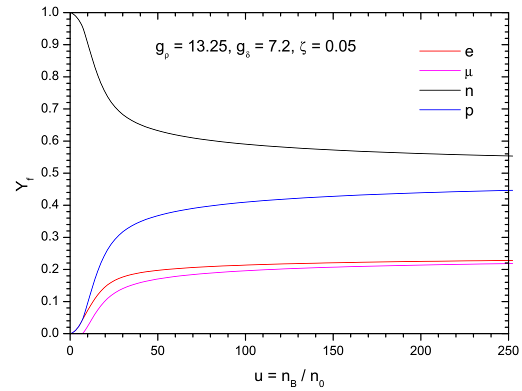

Figure 2. Particle concentrations in the neutron star matter.

Discussion

The obtained form of the equation of state (11) serves as an input for the Oppenheimer-Tolman-Volkoff equations. In the result hydrostatically stable configurations are calculated and the mass-radius relation can be drown. Fig.3 depicts some models of neutron stars which have been constructed for chosen parameter sets. In this figure ZM denotes the Zimanyj- Moszkowski model, QMF - the model for quark stars and GM3-the neutron stars model. For the given sequences of models the gravitational binding energy have been calculated and results are shown in Fig. 4. Analyzing the gravitational binding energy one can come to the conclusion that the configuration with δ meson is energetically favorable than the one without δ meson. However, for higher densities better results are obtained for the ZM and QMF models. Throughout the effective nucleon masses the meson δ alters nucleon chemical potentials what realizes in characteristic modification of the appearance, abundance and distributions of the individual species in neutron star matter. The inclusion of δ meson causes considerable nucleon mass splitting. In Fig.1 results are shown for chosen parameters marked as set III. (Table I). The appearance of additional nonlinear vector meson interactions produces also visible effects on the nucleon and lepton abundance. From Fig.3 one can see results obtained for the parameter set II.

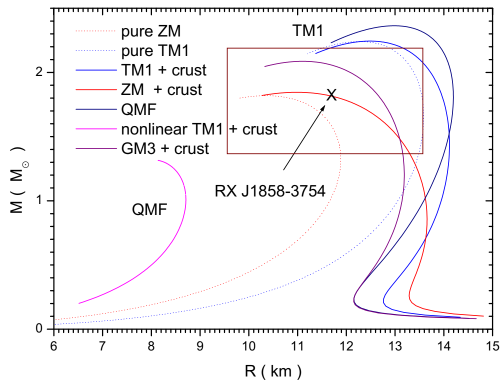

Figure 3. The mass radius relations for neutron stars in the RMF theory and for the quark star in the QMF theory.

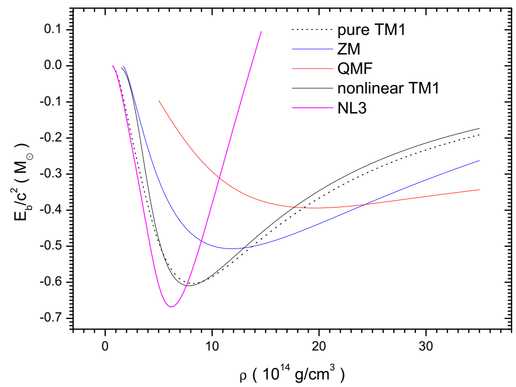

Figure 4. Gravitational binding E_g energy as a function of central density of neutron star and quark one.

Table 2

The neutron star properties in the nonlinear RMF model

parameter set	$M_{max}(M_\odot)$	$R_{max}(km)$	$M_{min}(M_\odot)$	$R_{min}(km)$	E_g
TM1	2.20	13.8	0.2	12.8	-0.617
GM3	2.17	12.9	0.2	12.2	-0.602
nonl. TM1	2.30	14.0	0.2	12.2	-0.586
RX J185635-3754	1.7 ± 0.4	11.4 ± 2	-	-	-

The analyzed form of the equation of state leads to the conclusion that the inclusion of δ meson and nonlinear vector meson interactions causes the softening of the equation of state and in the result the decrease of the neutron star parameters (Fig.3). Of special interest is the case which includes δ meson, the quartic ρ meson term and nonlinear vector meson interactions (set III in Table I). This case leads to the neutron star model with the value of the maximum radius $R_{max} = 14.0$ km. This result is consistent with recent observations [5] of RX J185635-3754.

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